Calculations of two integrals.
For problem by link:
https://www.linkedin.com/feed/update/urn:li:activity:6459729194510352384

* 1. $\int \arctan\left(1-\frac{1}{x}\right)dx = \left[\begin{array}{c} u' = 1; u = x \\ v = \arctan\left(1-\frac{1}{x}\right); v' = \dfrac{1}{2x^2-2x+1} \end{array}\right] =$

$x\arctan\left(1-\frac{1}{x}\right) - \int \frac{xdx}{2x^2-2x+1} =$

$x\arctan\left(1-\frac{1}{x}\right) - \frac{1}{2}\arctan(2x-1) - \frac{1}{4}\ln\left(x^2-x+\frac{1}{2}\right) + c$ and since

$1\cdot\arctan\left(1-\frac{1}{1}\right) = 0,\ \lim_{x\to 0}\left(x\cdot\arctan\left(1-\frac{1}{x}\right)\right) = 0, \ln\left(1^2-1+\frac{1}{2}\right) - \ln\left(0^2-0+\frac{1}{2}\right) = 0,$

$\arctan(2\cdot 1-1) - \arctan(2\cdot 0-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$ then $\int_0^1 \arctan\left(1-\frac{1}{x}\right)dx = -\frac{\pi}{4};$

2. $\int \arctan\left(\frac{1}{1-x}\right)dx = \left[\begin{array}{c} u' = 1; u = x \\ v = \arctan\left(\dfrac{1}{1-x}\right); v' = \dfrac{1}{x^2-2x+2} \end{array}\right] =$

$x\arctan\left(\frac{1}{1-x}\right) - \int \frac{xdx}{x^2-2x+2} = x\arctan\left(\frac{1}{1-x}\right) - \arctan(x-1) - \frac{1}{2}\ln(x^2-2x+2) + c$

and since $0\cdot\arctan\left(\frac{1}{1-0}\right) = 0,\ \lim_{x\to 1}\left(x\cdot\arctan\left(\frac{1}{1-x}\right)\right) = \frac{\pi}{2},$

$\arctan(1-1) - \arctan(0-1) = \frac{\pi}{4}, \ln(1^2-2\cdot 1+2) - \ln(0^2-2\cdot 0+2) = -\ln 2$

then $\int_0^1 \arctan\left(\frac{1}{1-x}\right)dx = \frac{\pi}{2} - \frac{\pi}{4} + \ln 2 = \frac{\pi}{4} + \frac{\ln 2}{2}$